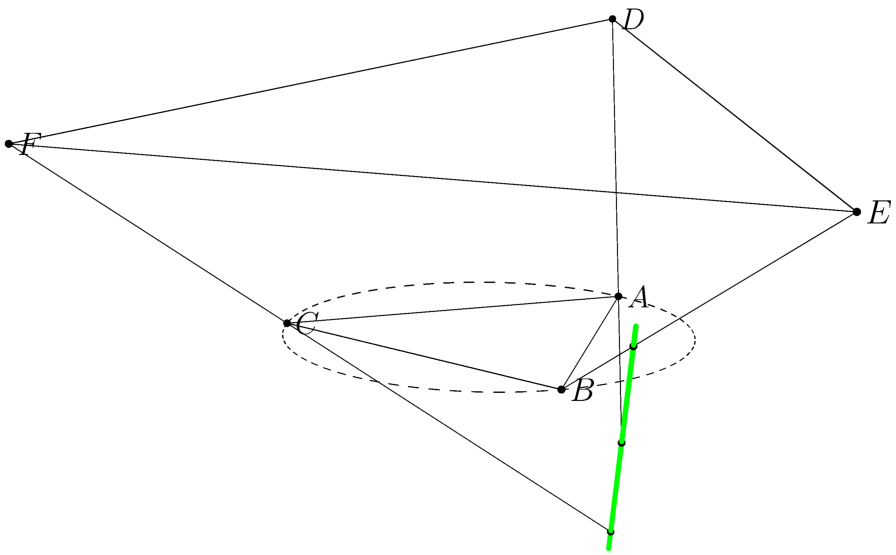
§ Problem Statement

In $\mathbb{R}^3$, tetrahedra $ABDE$, $BCEF$, and $CAFD$ are congruent. Line ℓ intersects $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$. Prove that ℓ meets (ABC).

§ Diagram



§ Solution

Claim 1. *The line through the centers of equilateral triangles ABC and DEF is perpendicular to each triangle.*

Proof. Fix equilateral triangle ABC and the shape of the congruent tetrahedra. Since the lengths AD, BD, and CD are fixed, there are only two choices for point D's location, which are symmetric about plane ABC. Additionally, the possible locations for E and F are rotations of the choices for D about the line through the center of ABC perpendicular to ABC. Since DEF must be equilateral, D, E, and F must all be on the same side of ABC. □

To finish the problem, let ℓ_{ax} be the line through the centers of equilateral triangles ABC and DEF, and let $\mathcal{H}$ be the hyperboloid of one sheet formed by rotating $\overline{AD}$ around ℓ_{ax}; the hyperboloid is not degenerate because $ABDE$ nonplanar implies ℓ_{ax} and $\overline{AD}$ do not intersect. Lines $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are contained in $\mathcal{H}$, so ℓ meets $\mathcal{H}$ at three distinct points. Since $\mathcal{H}$ is a quadric surface, this means ℓ is contained in $\mathcal{H}$. Now, ℓ must intersect the plane $\mathcal{P}$ through A, B, and C. Thus ℓ meets $(ABC) = \mathcal{P} \cap \mathcal{H}$.

§ Metadata

This problem was selected as Problem 5 of the 2026 AMM.

- Title: Line Meets Circle
- Author: Holden Mui
- Subject: geometry
- Description: line meets circle in 3D geometry problem
- Keywords: 3D, congruent quadrilaterals, line meets circle
- Difficulty: IMO Problem 2
- Collaborators: Linus Tang, Kathik Vedula, Andrew Wu, Albert Wang, Isaac Zhu
- Submission history: 2026 AMM
- Other credits: none