Patterns in the Air

A Collection of Insights on Math and Sound

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2026 American Masters of Mathematics

May 24, 2026

About me

- **High school:** math olympiads, alongside piano and composing.
- **College:** majored in math and music, minored in physics.
- **Now:** working in tech in San Francisco.
- I also write olympiad problems — including problems 3 and 5 on yesterday's contest!

Outline

1. Part A: Sound and Pitch

2. Part B: Ratios and Harmony

3. Part C: Build with boba straws!

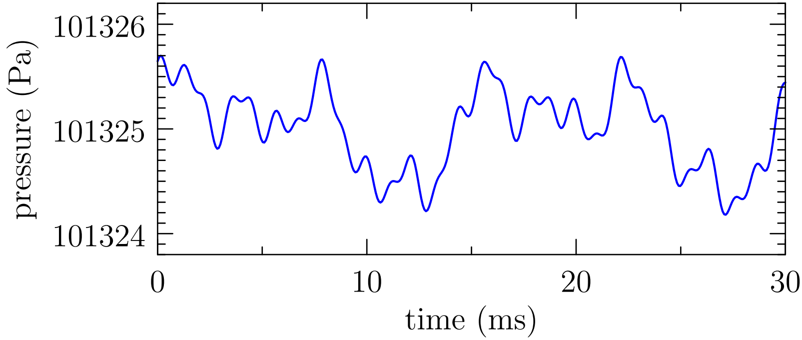
(with listening excerpts and video clips throughout!)

Part A

Sound and Pitch

What is sound?

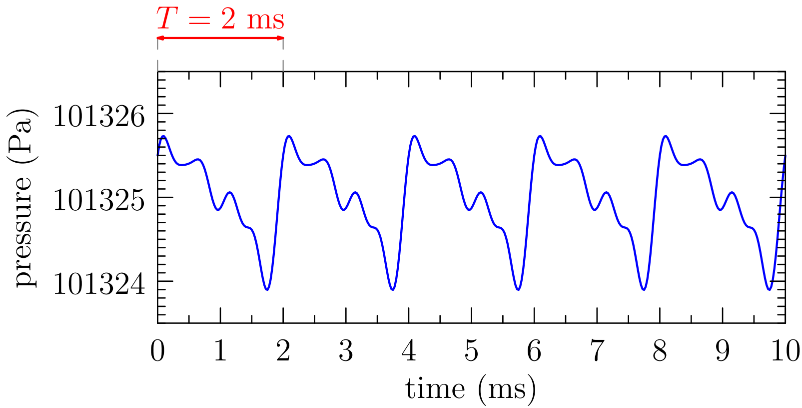
Sound is a *pressure wave* traveling through the air — a tiny variation around atmospheric pressure ($\approx$ 101,325 Pa).



[🔊 listen]

What is pitch?

What distinguishes a *pitch* from just a sound? [🔊 listen]

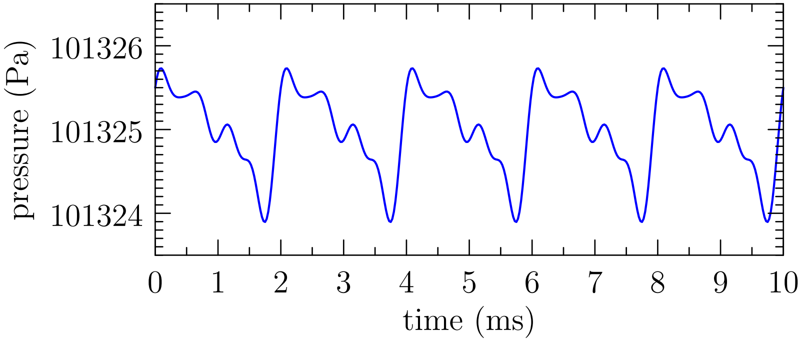


Pitches are formed from *periodic* pressure waves:

$$\text{pitch} = \frac{1}{T} = \frac{1}{0.002\,\text{s}} = 500\,\text{Hz}.$$

Detecting pitch

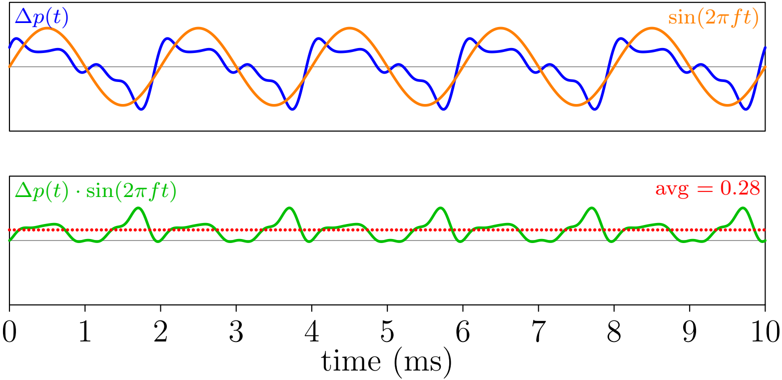
How can we tell, *algorithmically*, that this wave has frequency 500 Hz?



Idea: multiply by a sinusoidal wave of the suspected frequency, point by point, and see what the average is.

Try the right frequency

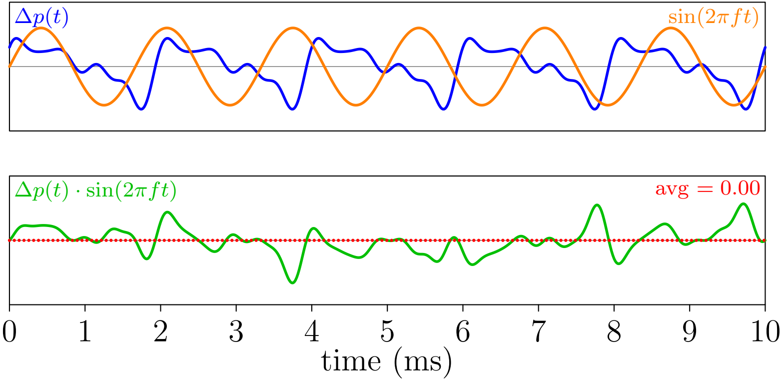
Multiply $\Delta p(t) = p(t) - p_{\text{atm}}$ by $\sin(2\pi ft)$ with $f = 500$ Hz:



Nonzero average product $\Rightarrow$ the wave *contains* a 500 Hz component.

Try a wrong frequency

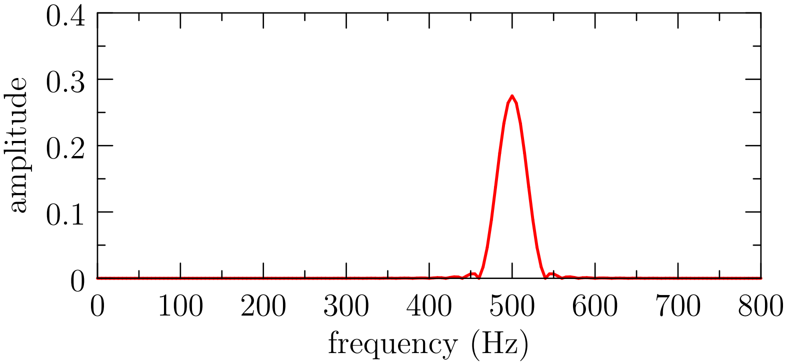
Now try $f = 600$ Hz instead:



Average ≈ 0 for all frequency-f sinusoids $\Rightarrow$ this frequency is *not* in the wave.

Just try every f

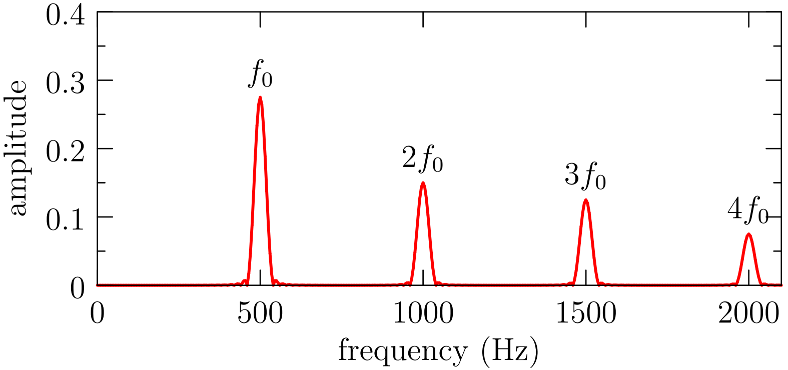
For each candidate frequency f, compute the average of $\Delta p(t)\sin(2\pi ft)$ — this is called the **Fourier transform** of $\Delta p(t)$.



The big peak at $f = 500$ Hz tells us that's the fundamental frequency.

Trying larger f

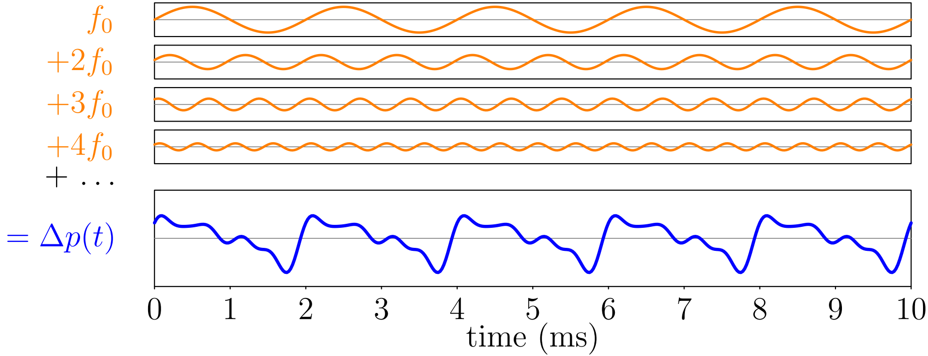
Extending this range further gives some surprises:



1000 Hz, 1500 Hz, 2000 Hz all match too — these peaks at kf_0 are the **harmonics** of the fundamental.

What does this mean?

Hidden inside every pitch are many sinusoidal waves. In fact, the original pressure wave is exactly the sum of those sinusoids:

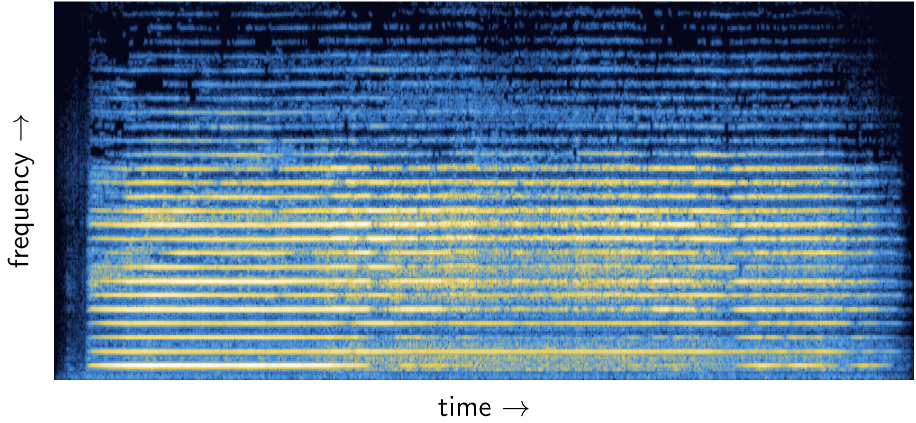


[🔊 listen]

Different instruments have different harmonic *amplitudes* — that's why a violin and a flute playing the same note sound different.

Spectrograms

A *spectrogram* plots frequency *vs.* time — one Fourier transform per short slice of audio.



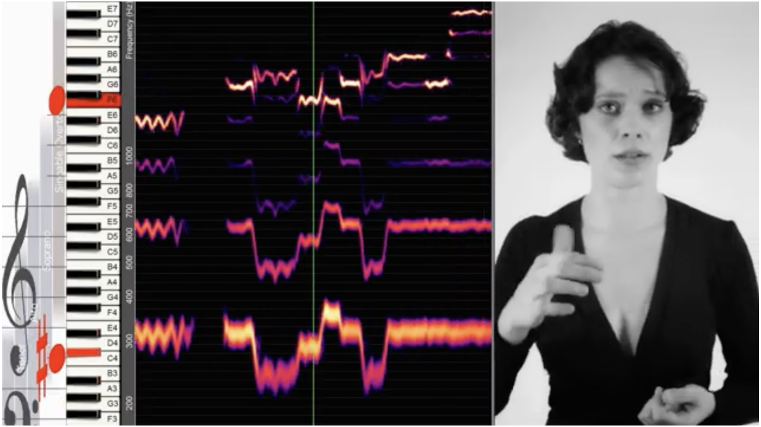
[🔊 listen]

These ideas explain musical illusions

- Polyphonic singing — singing two notes at once.
- Shepard tone — a sequence of notes that keeps going up forever.

Polyphonic singing

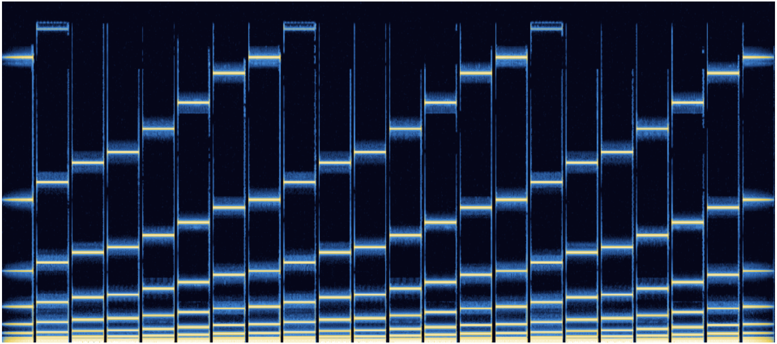
By shaping the mouth to single out one upper harmonic, the singer makes a single note sound like two voices.



[▶ play]

Shepard tone

A **Shepard tone** is an ascending scale that seems to climb forever. Several octaves play at once; as the top octave fades out, a new quiet one fades in at the bottom.



[🔊 listen to a Shepard tone]

Music inspired by harmonics and the spectrum

Three composers whose music draws on harmonics and the spectrum:

- Maurice Ravel (1875–1937)
- Gérard Grisey (1946–1998)
- Kaija Saariaho (1952–2023)

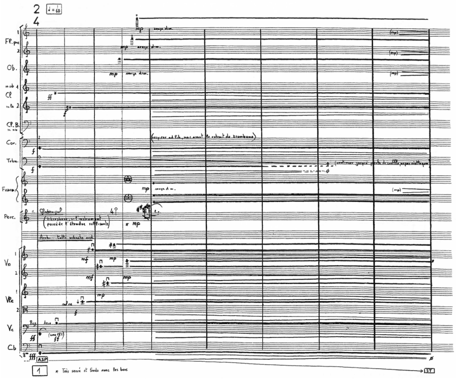
Maurice Ravel (1875–1937)

In *Bolero* (1928), Ravel orchestrates instruments to play different harmonics of one note — imitating a pipe organ.

[🔊 *Bolero* excerpt]

Gérard Grisey (1946–1998)

Gérard Grisey was a pioneer of **spectral music**. In *Partiels* (1975), he reconstructs the harmonic spectrum of a trombone E_2 using other instruments.



[*Partiels* excerpt]

Kaija Saariaho (1952–2023)

Kaija Saariaho was another famous spectralist composer. In *Lichtbogen* (1986), she takes a cello note morphing from a pure pitch into scratchy noise, captures its spectrum, stretches it across a 20-minute piece.

[🔊 *Lichtbogen* excerpt]

Aside: Subharmonics?

An object that naturally vibrates at frequency f_0 can sometimes be coaxed to vibrate at $f_0/2$, $f_0/3$, $\ldots$ instead — frequencies *below* the fundamental. A violin, for instance, can play notes below its lowest open string using special bowing techniques.

[🔊 listen to Mari Kimura's *Subharmonic Partita*]

Part B

Ratios and Harmony

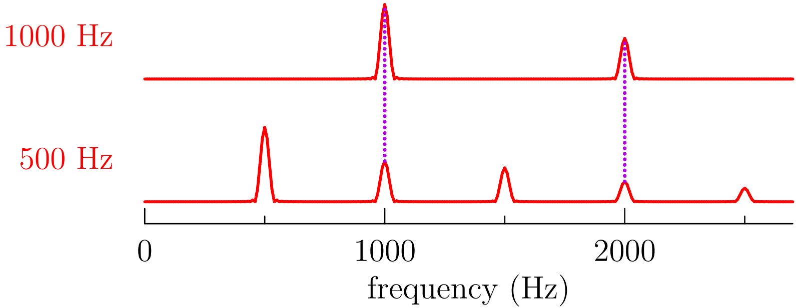
What makes some notes sound good together?

Listen to each pair — which one is more pleasing to the ear?

Pair 1:	[🔊 A]	[🔊 B]
Pair 2:	[🔊 A]	[🔊 B]
Pair 3:	[🔊 A]	[🔊 B]

Looking at their spectra

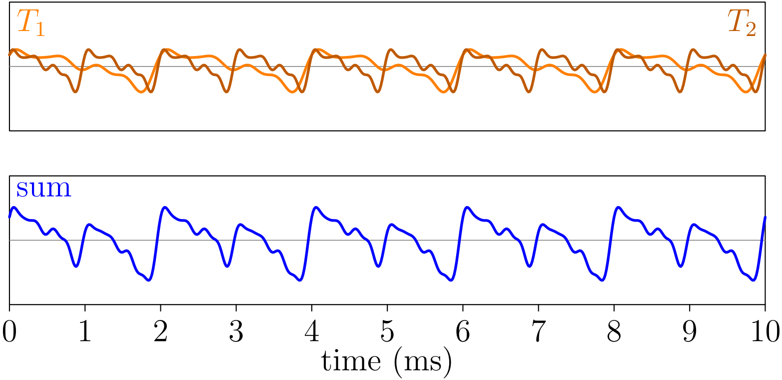
Two notes in ratio 2 : 1 — every harmonic of the higher note is already a harmonic of the lower:



[🔊 listen]

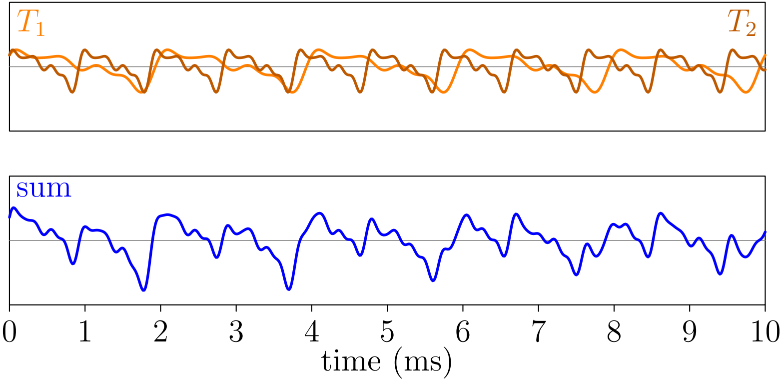
Equivalently, their sum has a small period.

Add two periodic waves with periods $T_1 = 2$ and $T_2 = 1$ ms (ratio 2:1):



Off-ratio: no clean period.

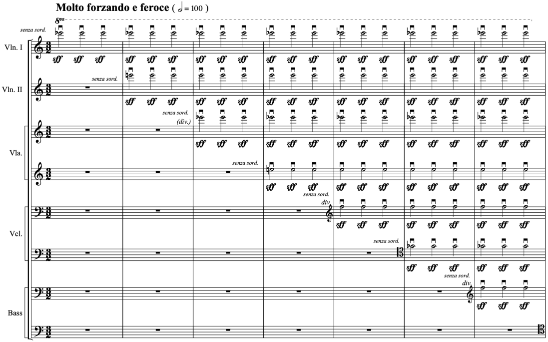
Now use $T_2 = 2/2.1$ ms instead of 1 (ratio 2.1:1 — not a ratio of small whole numbers):



[🔊 listen]

Example: *Psycho* (1960)

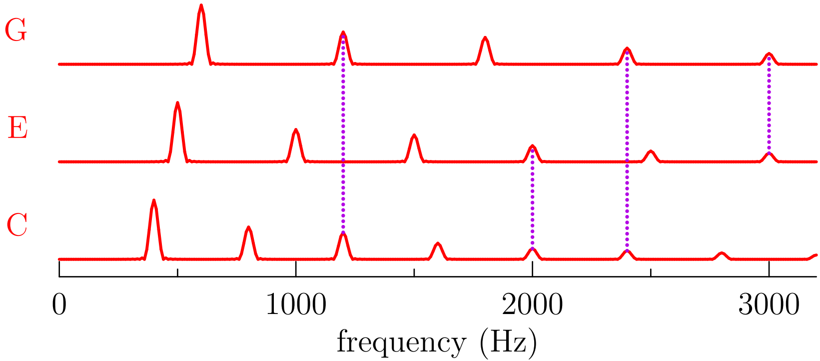
Bernard Herrmann's film score uses many *almost-octave* intervals — producing a deeply unsettling sound. The shower scene is the most famous example:



[🔊 shower scene]

The major chord

The most familiar consonant chord in Western music — three notes in ratio 4 : 5 : 6 (e.g. C : E : G).



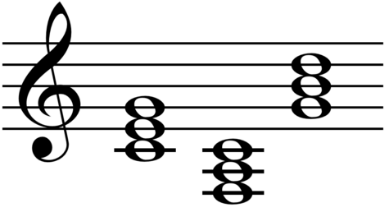
[🔊 listen]

Many low harmonics coincide ⇒ the chord sounds stable.

Building the major scale

The major scale is built by stacking three major chords (each in ratio $4:5:6$):

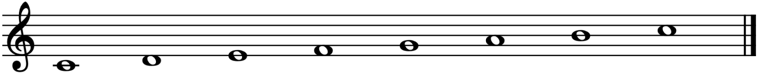
I:	[🔊]	C	E	G
		1	$\frac{5}{4}$	$\frac{3}{2}$
IV:	[🔊]	F	A	C
		$\frac{2}{3}$	$\frac{5}{6}$	1
V:	[🔊]	G	B	D
		$\frac{3}{2}$	$\frac{15}{8}$	$\frac{9}{4}$



The major scale

Combining all the notes, switching octaves if necessary:

C	D	E	F	G	A	B	C'
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2



[🔊 listen]

Western music

Nearly all of Western music is built on the major scale and its modes.

[🔊 Bach: *Goldberg Variations*]

Beating

When two notes are *almost* the same frequency,

you hear the sound **pulsating**. . .

[🔊 listen to beating]

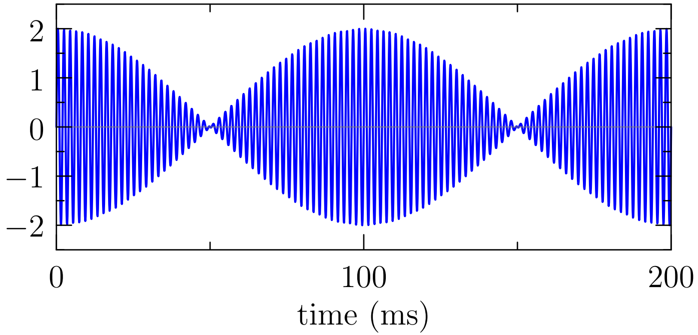
Musicians — especially string players and piano tuners — *tune* instruments by listening for the absence of beats.

The math of beating

Suppose you're listening to a 500 Hz and 510 Hz tone at the same time. By sum-to-product:

$$\sin(2\pi \cdot 500t) + \sin(2\pi \cdot 510t) = 2\sin(2\pi \cdot 505t)\cos(2\pi \cdot 5t).$$

A fast oscillation at the *average* (505 Hz) modulated by a slow 10 Hz envelope:



Music with beating: gamelan

Different cultures, different ideals. Western music prizes being in tune; gamelan deliberately tunes paired instruments apart for the shimmer of beats.

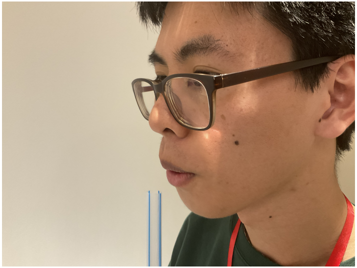
[▶ play]

Part C

Build with boba straws!

Let's build!

A boba straw is a tube open at both ends. Blow across the top to make a tone.



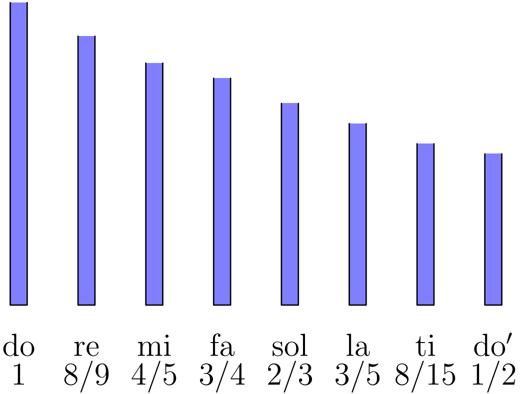
The fundamental frequency depends only on the length:

$$f = \frac{c}{2L}, \qquad c \approx 343 \text{ m/s}.$$

Pick a project on the next few slides — or invent your own.

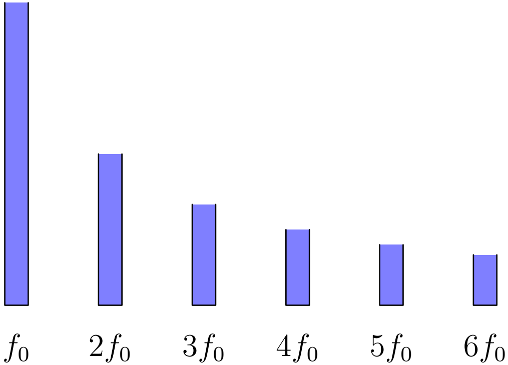
Project 1: Major scale pan flute

Cut your straws so their lengths are in the reciprocals of the just-intonation ratios (since $L \propto 1/f$):



Project 2: Spectral pipe

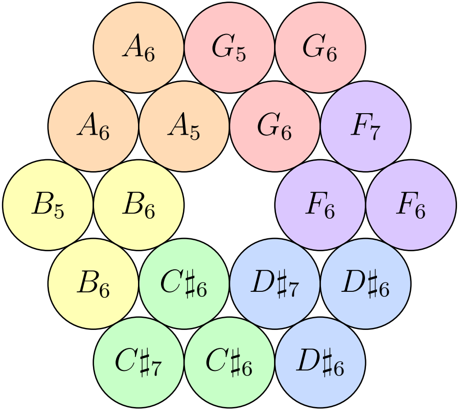
Tune your straws to the *harmonic series* of one fundamental — lengths $L, \frac{L}{2}, \frac{L}{3}, \ldots$ Use *multiple* straws of the same pitch to make that harmonic louder, then tape the whole bundle together.



Blown together, the bundle imitates a single rich note.

Project 3: Shepard tone

Pick any scale. Build it on several octaves; play in ascending order, fading the top octave out as a quieter one fades in at the bottom.



Project 4: Your own idea!

Be creative! Some ideas:

- boba trombone
- k-EDO pan flute for $k \neq 12$
- boba recorder

Thank you for listening!

Any questions?

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