

# Patterns in the Air

## A Collection of Insights on Math and Sound

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# About me

- **High school:** math olympiads, alongside piano and composing.
- **College:** majored in math and music, minored in physics.
- **Now:** working in tech in San Francisco.
- I also write olympiad problems — including problems 3 and 5 on yesterday's contest!



# Outline

- 1 Part A: Sound and Pitch
- 2 Part B: Ratios and Harmony
- 3 Part C: Build with boba straws!

*(with listening excerpts and video clips throughout!)*

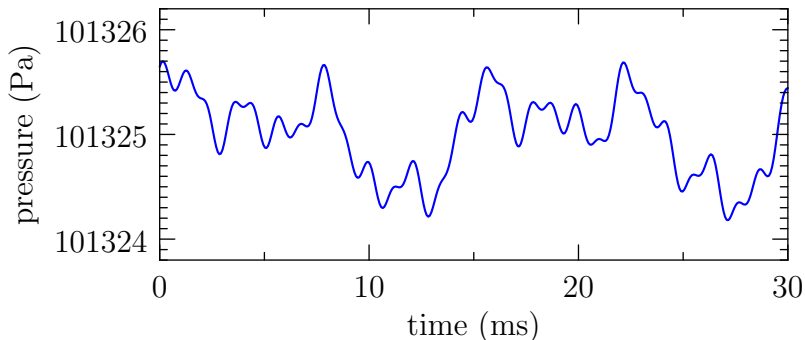
# **Part A**

Sound and Pitch



# What is sound?

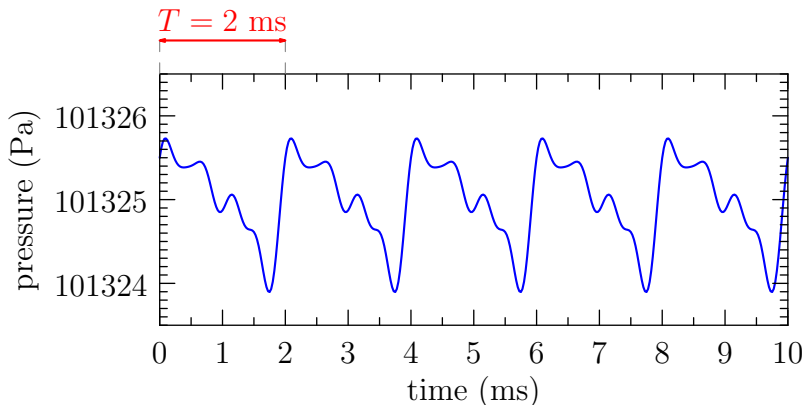
Sound is a *pressure wave* traveling through the air — a tiny variation around atmospheric pressure ( $\approx 101,325$  Pa).



 [listen](#)

# What is pitch?

What distinguishes a *pitch* from just a sound? [listen](#)

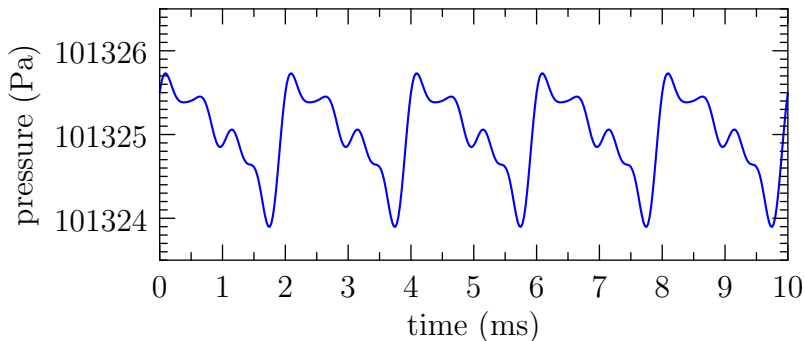


Pitches are formed from *periodic* pressure waves:

$$\text{pitch} = \frac{1}{T} = \frac{1}{0.002 \text{ s}} = 500 \text{ Hz.}$$

## Detecting pitch

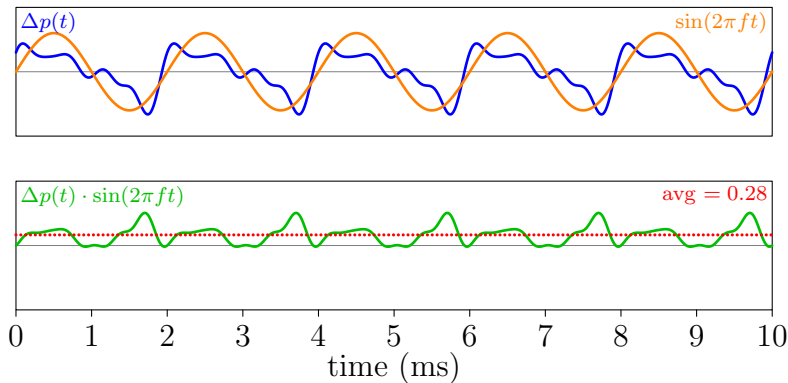
How can we tell, *algorithmically*, that this wave has frequency 500 Hz?



**Idea:** multiply by a sinusoidal wave of the suspected frequency, point by point, and see what the average is.

## Try the right frequency

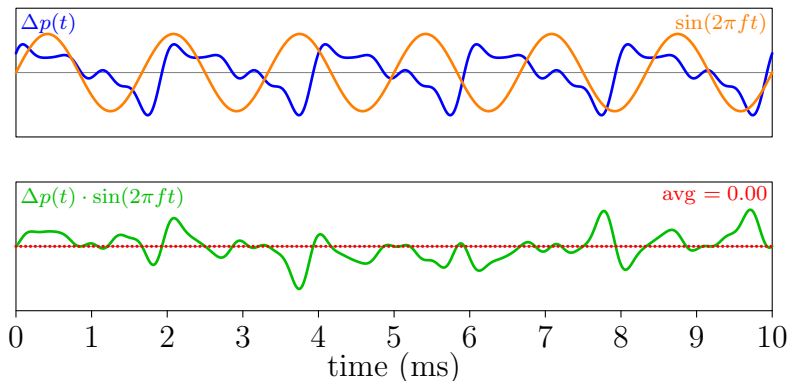
Multiply  $\Delta p(t) = p(t) - p_{\text{atm}}$  by  $\sin(2\pi ft)$  with  $f = 500$  Hz:



Nonzero average product  $\Rightarrow$  the wave *contains* a 500 Hz component.

## Try a wrong frequency

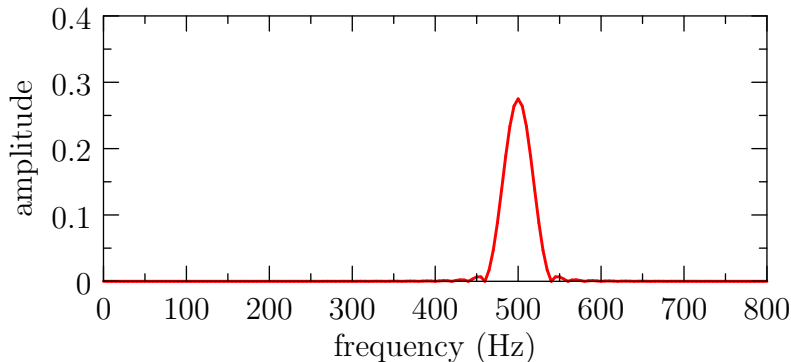
Now try  $f = 600$  Hz instead:



Average  $\approx 0$  for all frequency- $f$  sinusoids  $\Rightarrow$  this frequency is *not* in the wave.

## Just try every $f$

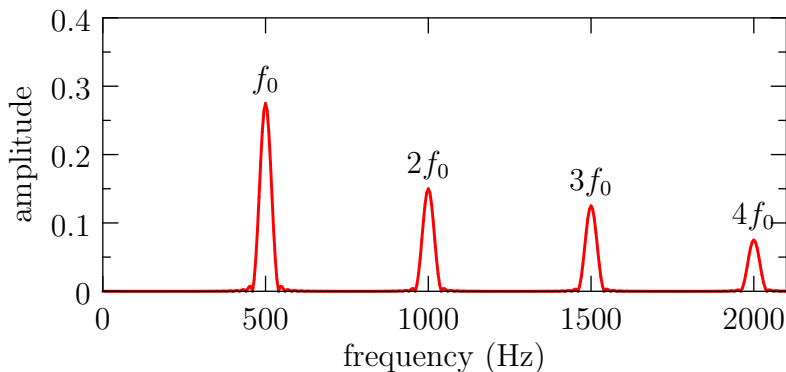
For each candidate frequency  $f$ , compute the average of  $\Delta p(t) \sin(2\pi ft)$  — this is called the **Fourier transform** of  $\Delta p(t)$ .



The big peak at  $f = 500$  Hz tells us that's the fundamental frequency.

## Trying larger $f$

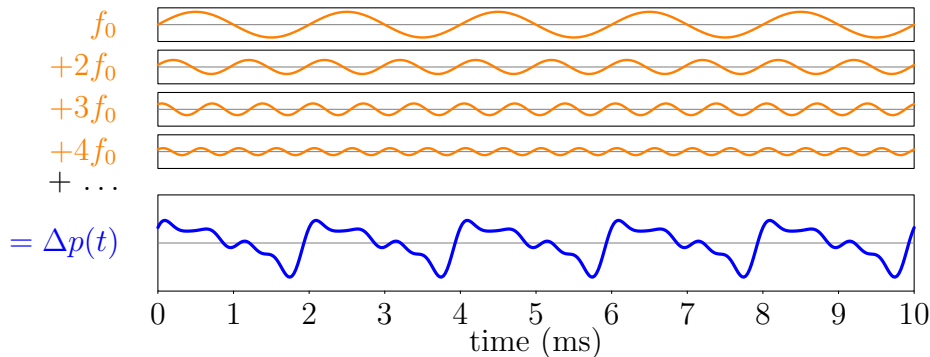
Extending this range further gives some surprises:



1000 Hz, 1500 Hz, 2000 Hz all match too — these peaks at  $kf_0$  are the **harmonics** of the fundamental.

## What does this mean?

Hidden inside every pitch are many sinusoidal waves. In fact, the original pressure wave is exactly the sum of those sinusoids:



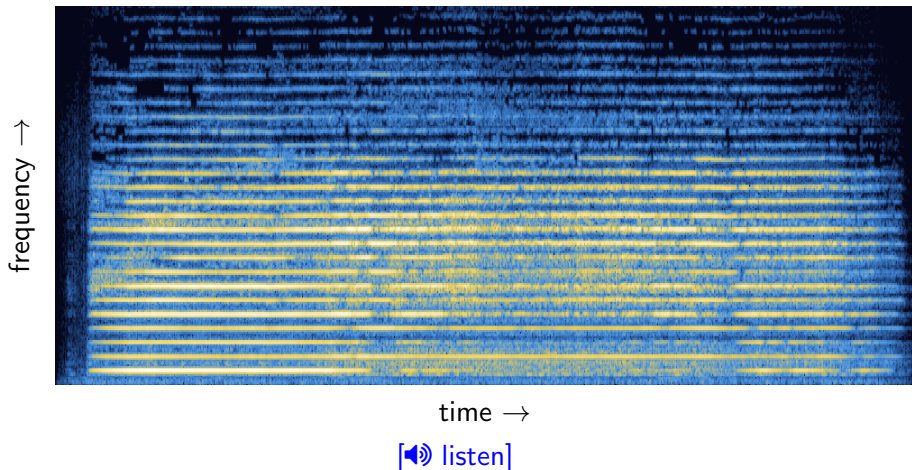
 [listen](#)

Different instruments have different harmonic *amplitudes* — that's why a violin and a flute playing the same note sound different.



# Spectrograms

A *spectrogram* plots frequency vs. time — one Fourier transform per short slice of audio.

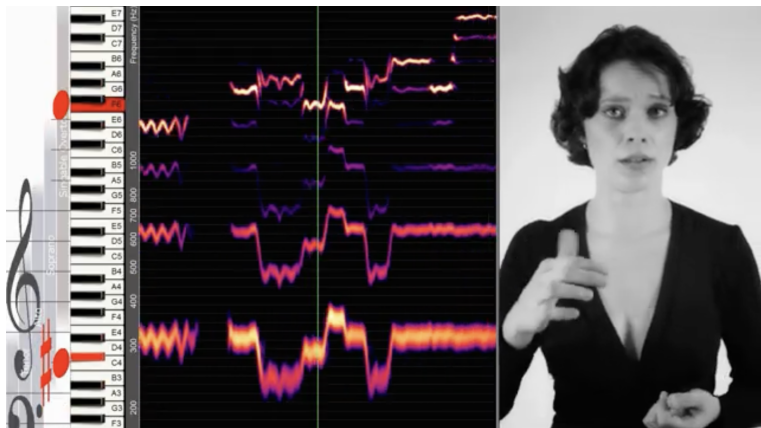


# These ideas explain musical illusions

- Polyphonic singing — singing two notes at once.
- Shepard tone — a sequence of notes that keeps going up forever.

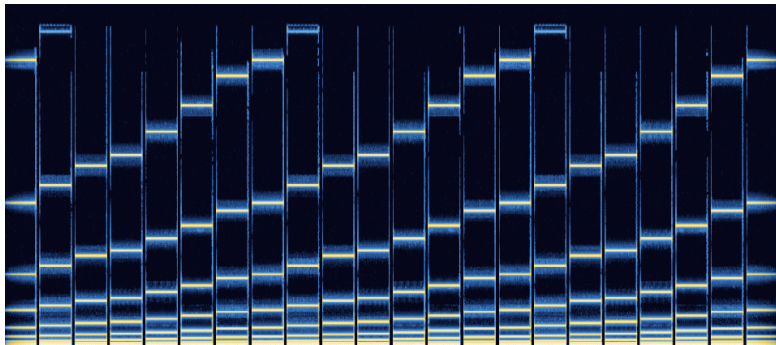
# Polyphonic singing

By shaping the mouth to single out one upper harmonic, the singer makes a single note sound like two voices.



# Shepard tone

A **Shepard tone** is an ascending scale that seems to climb forever. Several octaves play at once; as the top octave fades out, a new quiet one fades in at the bottom.



[ listen to a Shepard tone]

# Music inspired by harmonics and the spectrum

Three composers whose music draws on harmonics and the spectrum:

- Maurice Ravel (1875–1937)
- Gérard Grisey (1946–1998)
- Kaija Saariaho (1952–2023)

# Maurice Ravel (1875–1937)

In *Bolero* (1928), Ravel orchestrates instruments to play different harmonics of one note — imitating a pipe organ.

This musical score excerpt from Maurice Ravel's *Bolero* (measures 140-145) illustrates the concept of playing different harmonics of a single note. The score is for a full orchestra and includes the following parts:

- 1<sup>st</sup> Fl.**: Plays a melodic line starting in measure 141, marked *pp*.
- Fl.**: Plays a melodic line starting in measure 141, marked *pp*.
- Cl.B.**: Plays a melodic line starting in measure 141, marked *mf*.
- B<sup>tr</sup>**: Plays a melodic line starting in measure 141, marked *mf*.
- Sax. S<sup>ma</sup>**: Plays a melodic line starting in measure 141, marked *mf*.
- Cors.**: Plays a melodic line starting in measure 141, marked *mf*.
- Tromp.**: Plays a melodic line starting in measure 141, marked *mf*.
- Tamb.**: Plays a rhythmic pattern starting in measure 141, marked *mp*.
- Celesta**: Plays a melodic line starting in measure 141, marked *p*.

The score shows the instruments playing different harmonics of a single note, creating a rich, layered texture. The tempo is marked *Andante* and the key signature is one sharp (F#).

 *Bolero* excerpt]

# Gérard Grisey (1946–1998)

Gérard Grisey was a pioneer of **spectral music**. In *Partiels* (1975), he reconstructs the harmonic spectrum of a trombone E<sub>2</sub> using other instruments.

The image shows a handwritten musical score for the piece *Partiels* by Gérard Grisey. The score is written on multiple staves, each labeled with an instrument: Fl. (Flute), Cl. (Clarinet), Ob. (Oboe), Bsn. (Bassoon), Cor. (Cor Anglais), Trbn. (Trombone), Trpt. (Trumpet), Vcl. (Violin), Vcl. (Viola), Vcl. (Cello), and Cb. (Double Bass). The notation is dense and complex, featuring many notes, rests, and dynamic markings. The score is written in a style that is characteristic of spectral music, with a focus on the harmonic spectrum of the instruments. The score is divided into two systems, with the first system starting at measure 2 and the second system starting at measure 4. The score is written in a complex, dense notation with many notes and dynamic markings. The score is written in a style that is characteristic of spectral music, with a focus on the harmonic spectrum of the instruments. The score is divided into two systems, with the first system starting at measure 2 and the second system starting at measure 4. The score is written in a complex, dense notation with many notes and dynamic markings.

 *Partiels* excerpt

# Kaija Saariaho (1952–2023)

Kaija Saariaho was another famous spectralist composer. In *Lichtbogen* (1986), she takes a cello note morphing from a pure pitch into scratchy noise, captures its spectrum, stretches it across a 20-minute piece.

This image shows a musical score excerpt from Kaija Saariaho's *Lichtbogen*, measures 227 to 25. The score is for a large ensemble, including piccolo (picc), glockenspiel (glock), piano (pf), harp, violin I (vl I), violin II (vl II), viola (via), cello (vlc), and double bass (cb). The notation is complex, featuring many accidentals, dynamic markings (ppp, p, p (scarp), p (scarp)), and performance instructions like 'poco Ped.' and 'Cresc.'. The score is written in a single system with multiple staves. The bottom of the page shows a time signature of 5/4 and a tempo marking of 40. The score is marked with a circled 'U' and a '3' above the first measure.

 *Lichtbogen* excerpt



## Aside: Subharmonics?

An object that naturally vibrates at frequency  $f_0$  can sometimes be coaxed to vibrate at  $f_0/2$ ,  $f_0/3$ , ... instead — frequencies *below* the fundamental. A violin, for instance, can play notes below its lowest open string using special bowing techniques.

 [listen to Mari Kimura's \*Subharmonic Partita\*](#)

# Part B

Ratios and Harmony

# What makes some notes sound good together?

Listen to each pair — which one is more pleasing to the ear?

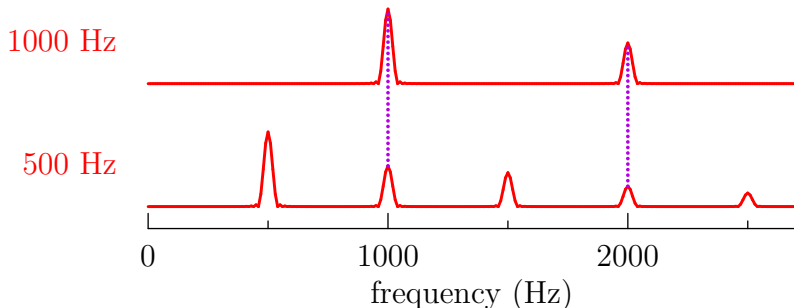
Pair 1:       A]       B]

Pair 2:       A]       B]

Pair 3:       A]       B]

## Looking at their spectra

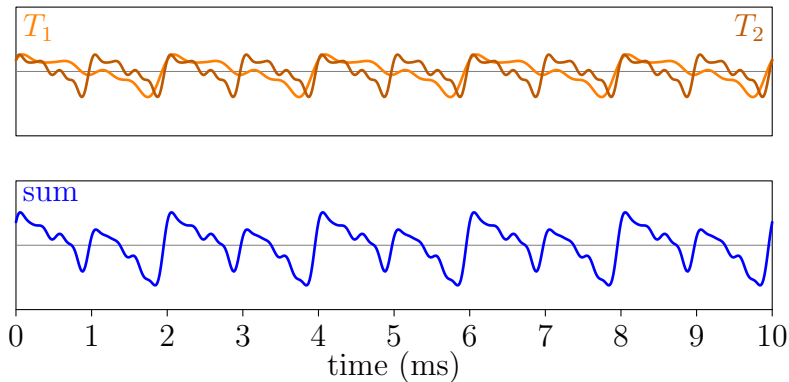
Two notes in ratio 2 : 1 — every harmonic of the higher note is already a harmonic of the lower:



[🔊 listen]

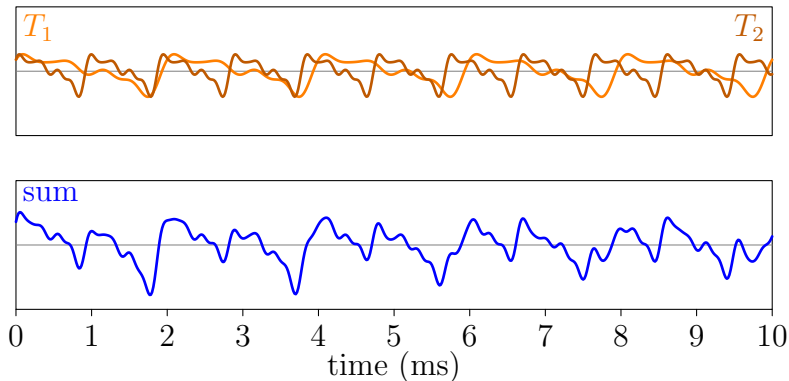
Equivalently, their sum has a small period.

Add two periodic waves with periods  $T_1 = 2$  and  $T_2 = 1$  ms (ratio 2:1):



## Off-ratio: no clean period.

Now use  $T_2 = 2/2.1$  ms instead of 1 (ratio 2.1:1 — not a ratio of small whole numbers):



[🔊] listen

## Example: *Psycho* (1960)

Bernard Herrmann's film score uses many *almost-octave* intervals — producing a deeply unsettling sound. The shower scene is the most famous example:

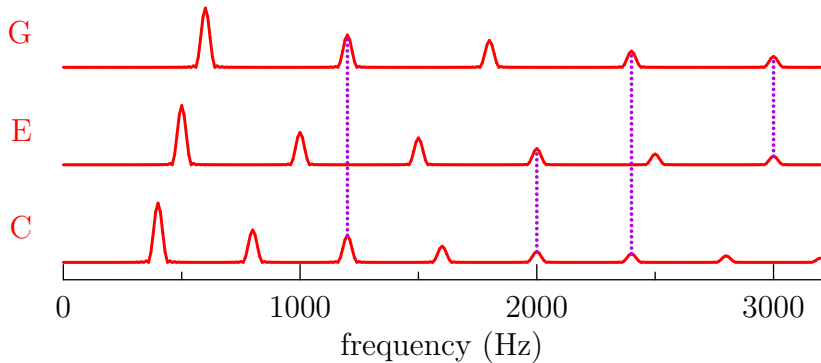
**Molto forzando e feroce** (♩ = 100)

The score is for a string ensemble (Violins I, Violins II, Viola, Violoncello, and Bass) and is marked **Molto forzando e feroce** (♩ = 100). The music is characterized by rapid, repetitive eighth-note patterns in the violins and a more active, descending line in the cello and bass. The score is written in 2/4 time and features several *senza sord.* (without mutes) markings, indicating a very loud and aggressive sound. The music is in the key of E major (three sharps).

 shower scene

## The major chord

The most familiar consonant chord in Western music — three notes in ratio 4 : 5 : 6 (e.g. C : E : G).




[\[🔊 listen\]](#)

Many low harmonics coincide  $\Rightarrow$  the chord sounds stable.



# Building the major scale

The major scale is built by stacking three major chords (each in ratio 4 : 5 : 6):

**I:**  C E G

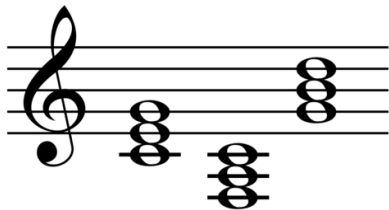
1  $\frac{5}{4}$   $\frac{3}{2}$

**IV:**  F A C

$\frac{2}{3}$   $\frac{5}{6}$  1

**V:**  G B D

$\frac{3}{2}$   $\frac{15}{8}$   $\frac{9}{4}$



# The major scale

Combining all the notes, switching octaves if necessary:

<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>A</i>	<i>B</i>	<i>C'</i>
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2



 listen]

# Western music

Nearly all of Western music is built on the major scale and its modes.



 Bach: *Goldberg Variations*

# Beating

When two notes are *almost* the same frequency,  
you hear the sound **pulsating** . . .

 [listen to beating](#)

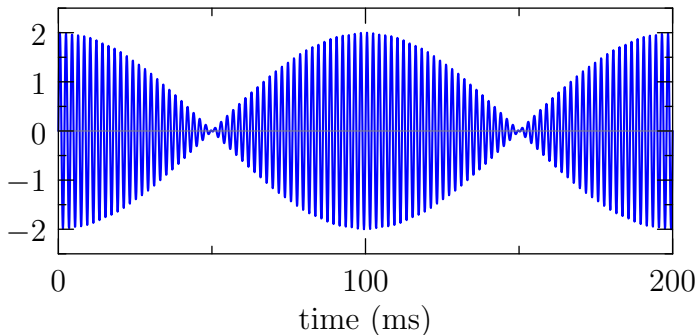
Musicians — especially string players and piano tuners — *tune* instruments by listening for the absence of beats.

## The math of beating

Suppose you're listening to a 500 Hz and 510 Hz tone at the same time. By sum-to-product:

$$\sin(2\pi \cdot 500t) + \sin(2\pi \cdot 510t) = 2 \sin(2\pi \cdot 505t) \cos(2\pi \cdot 5t).$$

A fast oscillation at the *average* (505 Hz) modulated by a slow 10 Hz envelope:



## Music with beating: gamelan

Different cultures, different ideals. Western music prizes being in tune; gamelan deliberately tunes paired instruments apart for the shimmer of beats.



# Part C

Build with boba straws!

## Let's build!

A boba straw is a tube open at both ends. Blow across the top to make a tone.



The fundamental frequency depends only on the length:

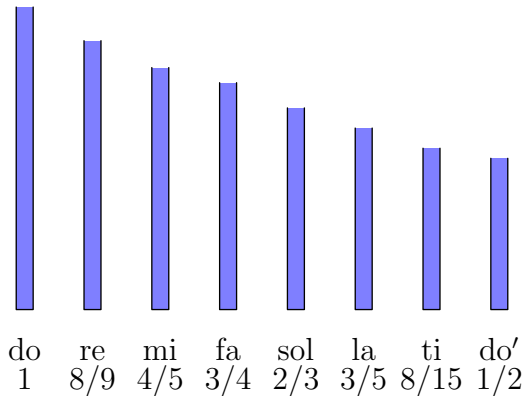
$$f = \frac{c}{2L}, \quad c \approx 343 \text{ m/s}.$$

Pick a project on the next few slides — or invent your own.



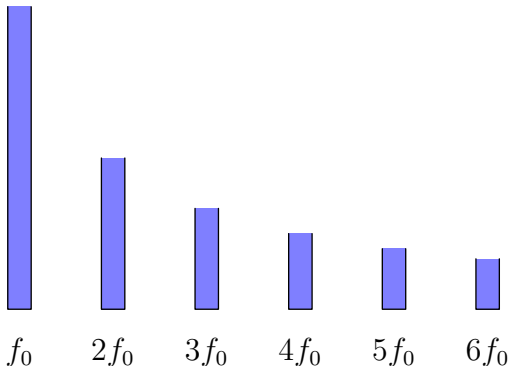
# Project 1: Major scale pan flute

Cut your straws so their lengths are in the reciprocals of the just-intonation ratios (since  $L \propto 1/f$ ):

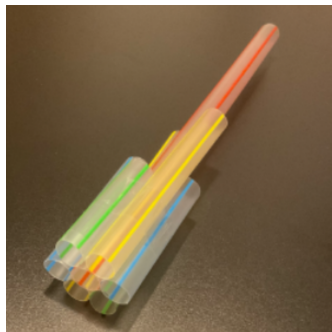


## Project 2: Spectral pipe

Tune your straws to the *harmonic series* of one fundamental — lengths  $L, \frac{L}{2}, \frac{L}{3}, \dots$ . Use *multiple* straws of the same pitch to make that harmonic louder, then tape the whole bundle together.

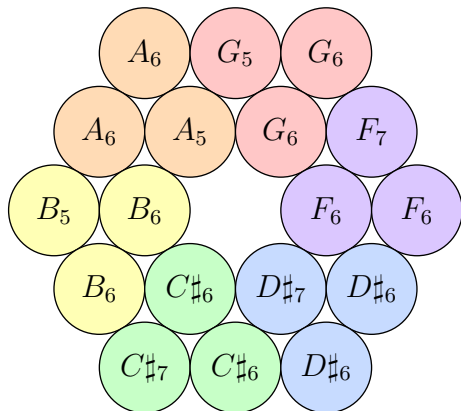


Blown together, the bundle imitates a single rich note.



## Project 3: Shepard tone

Pick any scale. Build it on several octaves; play in ascending order, fading the top octave out as a quieter one fades in at the bottom.



## Project 4: Your own idea!

Be creative! Some ideas:

- boba trombone
- $k$ -EDO pan flute for  $k \neq 12$
- boba recorder

# Thank you for listening!

Any questions?

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